

May, 16, 2002. Final Exam for S156: Applied time series analysis

Name: _____

Below, $\{a_t\}$ denotes a sequence of iid random variables with zero mean and finite variance $\sigma_a^2 > 0$.

1. Let Z_1, Z_2, \dots, Z_{100} be an MA(2) process: $Z_t = (1 + 0.5B - 0.2B^2)a_t$. Listed below are the last two observed and fitted values:

	$t = 99$	$t = 100$
Z_t	3	1
$\hat{Z}_{t-1}(1)$	1	0

- (a) Predict Z_{101}, Z_{102} , and Z_{103} .

- (b) Find the variance of the prediction errors in part (a). (Assume the noise variance to be 1.)

- (c) Compute 95% prediction intervals for Z_{101}, Z_{102} , and Z_{103} . State any distributional assumption of the a 's needed for the validity of the prediction intervals.

2. From a series of length 100, we have computed $r_1 = 0.8, r_2 = 0.4, r_3 = 0.2$, and $s_Z^2 = 1$.

(a) Assume that an ARMA(1,2) model: $Z_t = \phi_1 Z_{t-1} + a_t + 0.8a_{t-2}$ is appropriate for this data set. Find a consistent estimator of ϕ , using the method of moment.

(b) Use the method of moment to derive an estimator for σ_a^2 .

3. This question concerns the empirical identification of ARIMA models.

(a) The sample ACFs, PACFs, and EACF for a series are given below (n=100):

	lag									
	1	2	3	4	5	6	7			
ACF	-0.616	0.214	0.013	-0.021	0.101	-0.189	0.048	0.101	-0.095	-0.042
PACF	-0.616	-0.265	0.029	0.102	0.223	-0.07	-0.289	-0.056	0.109	-0.018

AR/MA

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	"x"	"x"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"
1	"x"	"x"	"x"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"
2	"o"	"x"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"
3	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"
4	"x"	"o"	"x"	"o"	"o"	"x"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"
5	"x"	"x"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"
6	"x"	"x"	"o"	"o"	"o"	"x"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"
7	"x"	"x"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"	"o"

Based on the above statistics, what ARIMA model(s) would you consider for the series? Explain your reasoning for credits.

- (b) The time-series plot of 100 quarterly observations of a variable W , and the acf of W , $(1 - B)W$ and $(1 - b^4)W$ are shown in Fig. 1. Also the sample variances of W , $(1 - B)W$ and $(1 - b^4)W$ equal 1.703, 1.371 and 1.665 respectively. Should the data be differenced? If so, would you do regular or seasonal differencing? What ARIMA model would you consider for W . Explain your answers for credits.

4. An AR(1) and an MA(2) model were fitted to a time series with 100 observations. Below are the two fitted models

```
arima(x = Z, order = c(0, 0, 2))
```

Coefficients:

	ma1	ma2	intercept
	1.0595	0.7507	-0.4704
s.e.	0.0738	0.0581	0.4104

sigma² estimated as 2.172: log likelihood = -181.73, aic = 371.46

```
arima(x = Z, order = c(1, 0, 0))
```

Coefficients:

	ar1	intercept
	0.9368	0.0662
s.e.	0.0319	1.6024

sigma² estimated as 1.299: log likelihood = -156.01, aic = 318.01

(a) Which one of the two models provide better fit to the data. Explain your answer to get credits.

(b) Construct a 95% confidence interval for the AR(1) coefficient.

(c) Figures 2 and 3 show some of the diagnostics check with the fitted AR(1) model. Comment on goodness of fit of the AR(1) model. In particular, comment on whether the residuals (i) have any outliers, (ii) conform to the iid normal assumption, and (iii) are of constant variance. If you judge the fitted AR(1) model inadequate, that is, the fit is poor, what model would you consider for the data, in view of the information from the residuals. You must explain your comments to get credits.

5. Consider the following model:

$$Z_t = \phi_0 + \phi_t Z_{t-4} + a_t,$$

where

$$\phi_t = \begin{cases} \beta_0, & \text{if } t = 4m \text{ for some integer } m \\ \beta_1, & \text{if } t = 4m + 1 \text{ for some integer } m \\ \beta_2, & \text{if } t = 4m + 2 \text{ for some integer } m \\ \beta_3, & \text{if } t = 4m + 3 \text{ for some integer } m. \end{cases}$$

(a) Show that if the β 's are not identical, then the process $\{Z_t\}$ is not stationary.

(b) Show that if $|\beta_0| < 1$, then the process $\{Z_{4t}, t = 0, 1, 2, 3, \dots\}$ is stationary.

- (c) Suppose that $|\beta_i| < 1, i = 0, 1, 2, 3$. Given data Z_0, Z_1, \dots, Z_n , explain how you would check the assumption that the β 's are identical. You have to state the test statistic and how to check the significance of the test.

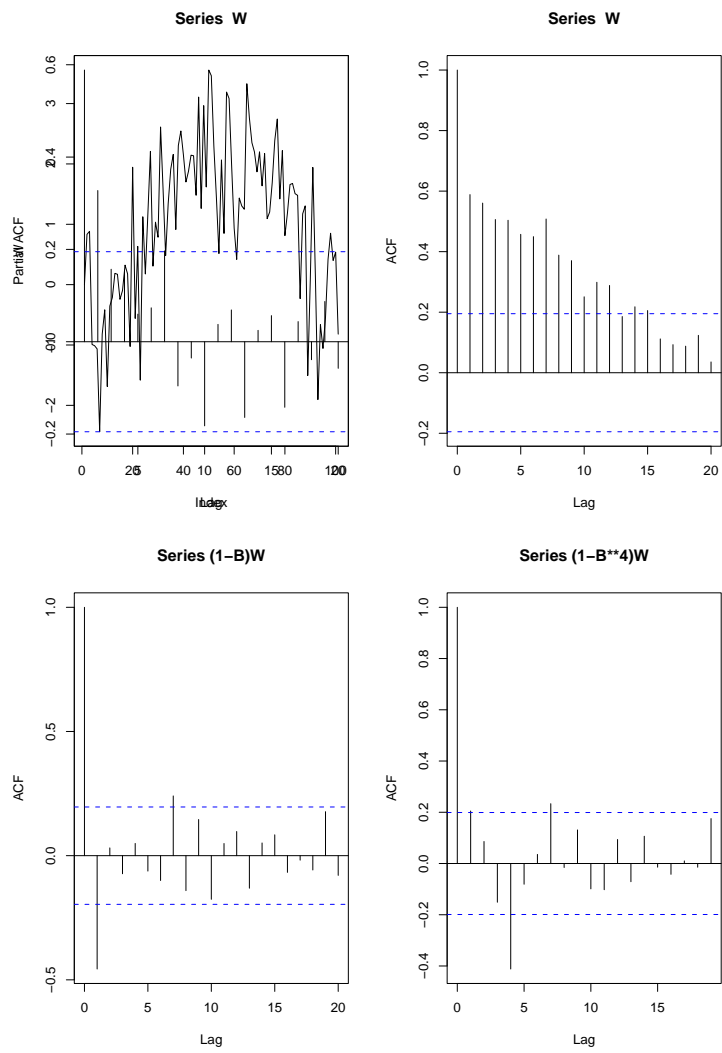


Figure 1:

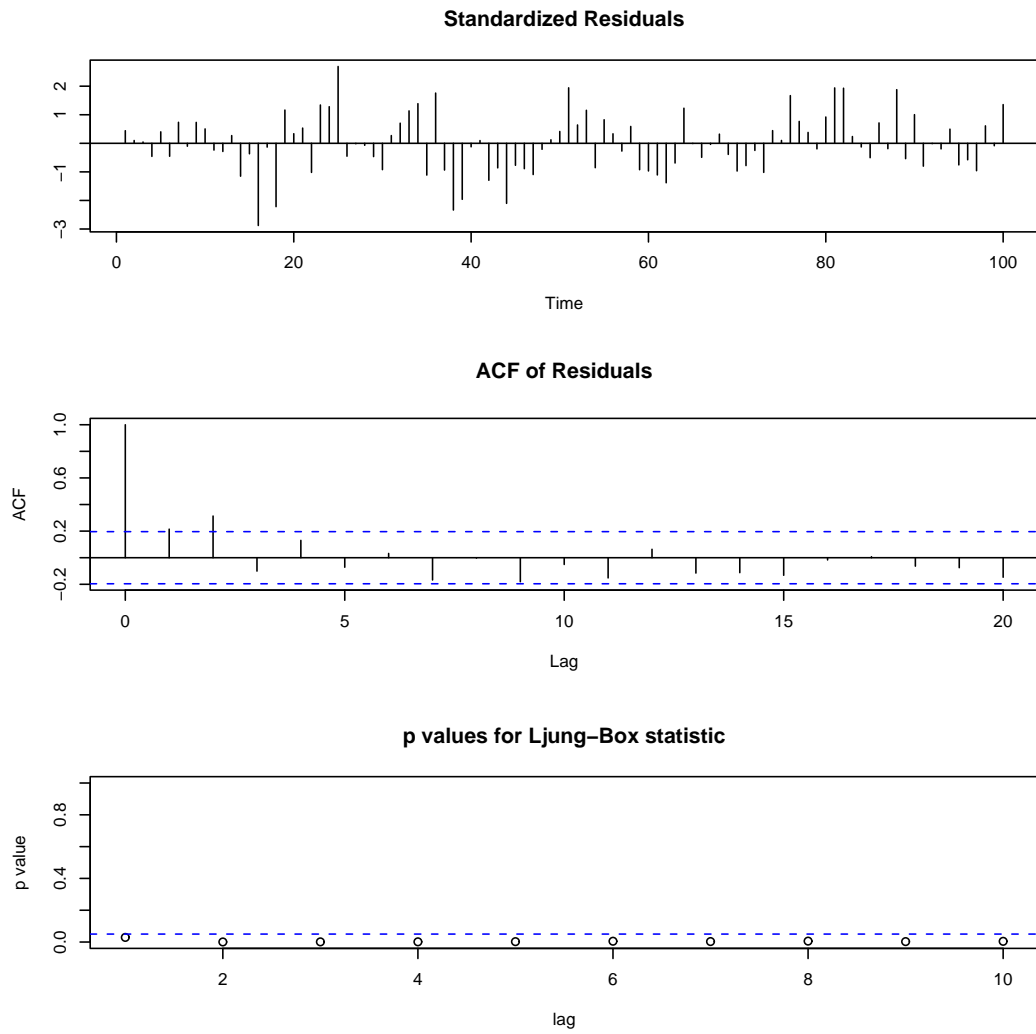


Figure 2: The upper figure shows the standardized residual plot, the middle figure shows the ACF of the residuals, and the bottom figure shows the p-values of the Box-Ljung statistics.

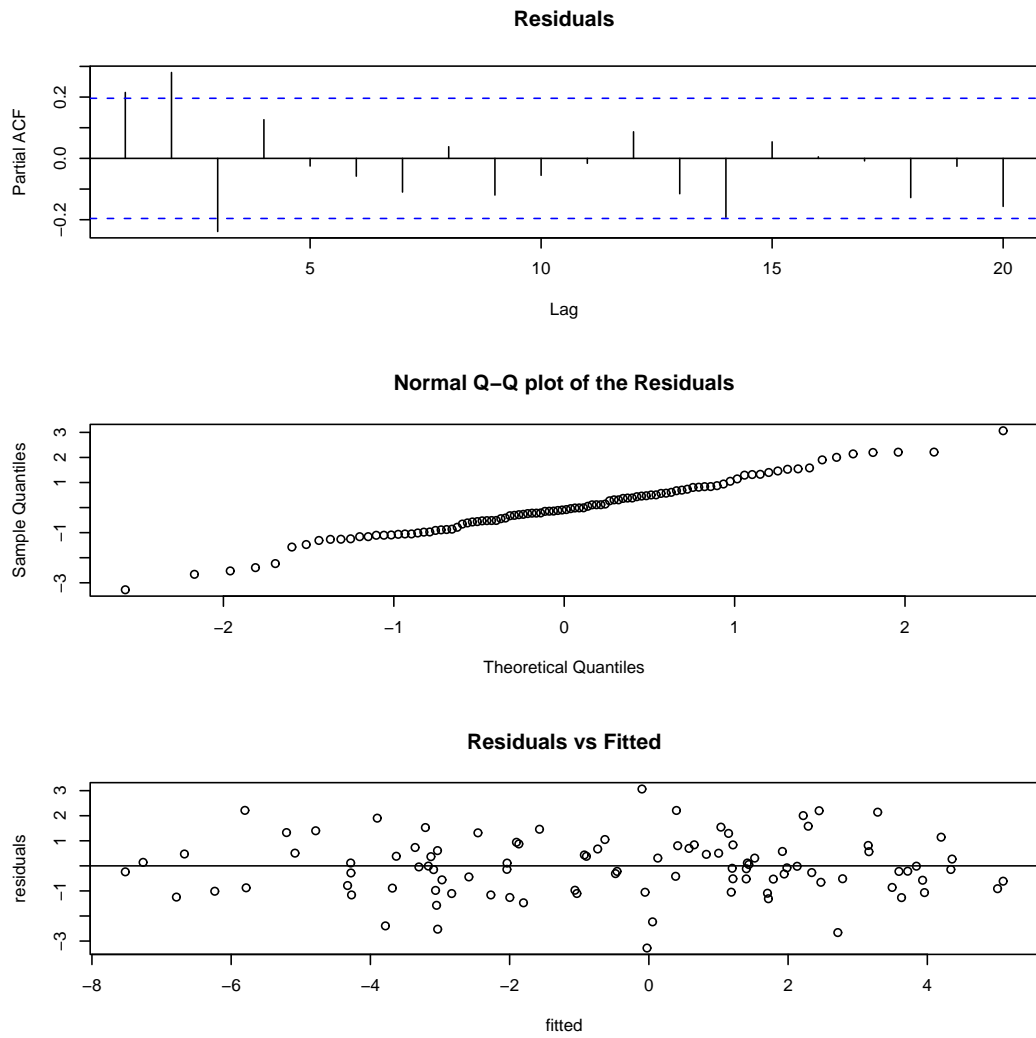


Figure 3: The upper figure shows the pacf of the residuals, the middle figure is the q-q normal plot of the residuals and the bottom figure shows the residuals vs fitted plot.