

BIBD: Interblock information

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Recall when we did an intra-block analysis, we considered blocks to be fixed and removed the block effects by subtracting the block means from the data.

But, it turns out that those block means that were subtracted from the data also contain information about treatment differences.

	TRT		
Block 1	1	2	3
Block 2	1	2	4
Block 3	1	3	4
Block 4	2	3	4

Interblock analysis

We now consider blocks to be random and get another set of estimates for the α_i 's called the interblock estimates (which are independent of the earlier found intrablock estimates).

$$y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}$$

α_i : trt
 β_j : block
 ϵ_{ij} : ind.

$\beta_j \stackrel{iid}{\sim} N(0, \sigma_b^2)$
 $\epsilon_{ij} \stackrel{iid}{\sim} N(0, \sigma_e^2)$

Block total for block j

$$Y_{\cdot j} = k\mu + \sum_{i=1}^g n_{ij} \alpha_i + k\beta_j + \sum_{i=1}^g n_{ij} \epsilon_{ij}$$

Sum across k trts in block j

indicator variable $n_{ij} = \begin{cases} 1 & \text{if trt } i \text{ in block } j \\ 0 & \text{o.w.} \end{cases}$

Writing in another form:

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$$Y_{\cdot j} = k\mu + n_{1j}\alpha_1 + n_{2j}\alpha_2 + \dots + n_{gj}\alpha_g + (\text{error}_j)$$

and this looks like a linear regression with g predictors

Some tedious algebra and manipulations leads to the least squares estimates of

$$\begin{aligned}\tilde{\mu} &= \bar{Y}_{\cdot\cdot} \\ \tilde{\alpha}_i &= \frac{\sum_{j=1}^b n_{ij} Y_{\cdot j} - r k \tilde{\mu}}{r - \lambda}\end{aligned}$$

which are the interblock estimates.

these estimates are based on block totals (disregarding within block structure). this is information picked-up from block-to-block.

(Recall the intrablock analysis was based on within block information as block effects were removed.)

What are the contrast estimates and variances? (in the interblock analysis)

$$\begin{aligned}\tilde{\gamma} &= \tilde{\alpha}_i - \tilde{\alpha}_j \\ \text{Var}(\tilde{\gamma}) &= \frac{(k^2 \sigma_\beta^2 + k\sigma^2)}{r - \lambda}\end{aligned}$$

In general

$$\text{Var} \left(\sum_{i=1}^g C_i \tilde{Q}_i \right) = (k^2 \sigma_\beta^2 + k \sigma^2) \sum_{i=1}^g \frac{C_i^2}{r-\lambda}$$

(3)

the estimator for σ^2 comes from the intrablock analysis:

$$\hat{\sigma}^2 = \text{MSE} \quad (\text{from fixed TRT \& fixed blocks})$$

The estimator for σ_β^2 involves doing something unusual and that's adjusting blocks for treatments (only place will see this)

$$\hat{\sigma}_\beta^2 = \frac{(b-1) [MS_{\text{Block(treatment)}} - \text{MSE}]}{N-g}$$

$$\text{where } E[MS_{\text{Block(treatment)}}] = \sigma^2 + \frac{(N-g)\sigma_\beta^2}{b-1}$$

We now have 2 distinct estimators for treatment effects

the two estimators are uncorrelated $\text{Cov}(\hat{Q}_i, \tilde{Q}_i) = 0$

Can we bring these 2 estimators together?

YES. Combining them is called

"Recovery of interblock information."

We'll use a weighted average to form the combined estimator such as $\bar{Q} = w\hat{Q}_i + (1-w)\tilde{Q}_i$ and will use weights based on variances. (4)

$$\bar{Q} = \frac{\frac{1}{\text{var}(\hat{Q}_i)} \cdot \hat{Q}_i + \frac{1}{\text{var}(\tilde{Q}_i)} \cdot \tilde{Q}_i}{\left(\frac{1}{\text{var}(\hat{Q}_i)} + \frac{1}{\text{var}(\tilde{Q}_i)} \right)}$$

and

$$\text{Var}(\bar{Q}) = \frac{1}{\left(\frac{1}{\text{var}(\hat{Q}_i)} + \frac{1}{\text{var}(\tilde{Q}_i)} \right)}$$

We don't gain a lot if $\sigma_\beta^2 \gg \sigma^2$.

When you use PROC MIXED in SAS and let blocks be random, it does the combined analysis.