

22s:039 Probability and Statistics for Engineers
Exam 1A
Fall 2009

Thursday, September 24, 8 - 9:30pm

100 possible points

Student Name _____
Section [letter/#] _____
Section [day/time] _____

Instructions:

- 1) Make sure you have the correct number of pages. There are 8 pages and 14 questions.
Each question is worth 7 points (you get 2 points for putting your section above).
- 2) Please use a pencil.
- 3) Questions can be clarified, but no hints will be provided.
- 4) You have 1.5 hours to complete the exam.
- 5) Show your student ID to a TA or professor when handing the exam in.
- 6) Please hand-in your formula sheet with your exam (staple to back).
- 7) You may begin, good luck.

Questions 1-2.

Let X be the random variable representing the number of scratches on a newly finished piece of furniture. Suppose X has the probability mass function $f(x)$ below:

X	0	1	2	3
$f(x)$	0.73	0.15	0.08	0.04

1. Calculate $E(X)$. [7 pts]

a) 0.83

b) 2.28

c) 1.16

d) 0.43

e) None of the answers is correct to the second decimal place.

2. Calculate $V(X)$. [7 pts]

a) 0.6451

b) 1.2455

c) 0.9823

d) 0.6046

e) None of the answers is correct to the fourth decimal place..

Question 3.

3. In a chemical process, experiments using five different amounts of calcium carbonate are performed. The results from each of these may or may not be further reduced in a second step using four different calcium carbonate amounts. Each of these results may or may not be further reduced in a third step using three different amounts of calcium carbonate.

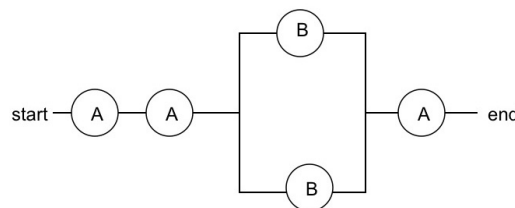
If all outcomes are equally likely, what is the probability that the best result is obtained from an experiment that uses two steps? [7 pts]

Circle your answer:

- a) 0.0589
- b) 0.4000
- c) 0.7059
- d) 0.2353
- e) None of the answers is correct to the fourth decimal place.

Question 4.

4. Relays used in the construction of electrical circuits function independent of each other. Two types of relays are used in the diagram below. Type A and Type B. Type A functions properly with a probability of 0.8. Type B functions properly with a probability of 0.6.



What is the probability that the circuit functions properly? (i.e. current flows from start to end.) [7 pts]

- a) 0.5699
- b) 0.4301
- c) 0.0051
- d) 0.1843
- e) None of the answers is correct to the fourth decimal place.

Questions 5-6.

Samples of rejuvenated mitochondria are mutated in 10% of cases. Suppose 5 samples are studied, and they can be considered to be independent for mutation.

5. Determine the probability that exactly four samples are mutated. [7 pts]

- a) 0.32805
- b) 9×10^{-5}
- c) 1×10^{-4}
- d) 0.00045
- e) None of the answers is correct.

6. Determine the probability that at most four samples are mutated. [7 pts]

- a) 0.00001
- b) 0.00045
- c) 0.99999
- d) 0.65610
- e) None of the answers is correct to the fifth decimal place.

Question 7.

7. In one day, an insurance company adjuster received 13 new accident claims. Four house claims, six car claims, two truck claims, and one pet injury claim. Assuming he will follow-up on these claims in random order, what is the probability that NONE of the house claims are among the first six he follows up on? [7 pts]
Circle your answer:

- a) 0.0490
- b) 0.0210
- c) 0.9510
- d) 0.9790
- e) None of the answers is correct to the fourth decimal place.

Questions 8-9.

8. Suppose we have a population of people living on an island, and we categorize them as democrat (D), republican (R), or other (O). Based on the number of people in each category, we have $P(D)=0.36$, $P(R)=0.44$, and $P(O)=0.20$.

We are actually interested in the housing values on the island, and we will let H be the event that a person on the island has a house with a high value. We know $P(H|D)=0.25$, $P(H|R)=0.56$, and $P(H|O)=0.65$. What is $P(H)$? [7 pts]

- a) 0.4867
- b) 0.4416
- c) 0.4664
- d) 0.6998
- e) None of the answers is correct to the fourth decimal place.

9. If you draw a person at random from those people with a high housing value, what is the probability of picking someone who is a democrat? [7 pts]
- a) 0.3600
 - b) 0.2787
 - c) 0.5283
 - d) 0.1930
 - e) None of the answers is correct to the fourth decimal place.

Question 10.

10. The cumulative distribution function is shown below for the random variable X.

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.05 & 0 \leq x < 0.25 \\ 0.50 & 0.25 \leq x < 0.75 \\ 0.95 & 0.75 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

What is the $P(X = 0.5)$? [7 pts]

- a) 0
- b) 0.50
- c) 0.45
- d) 0.75
- e) None of the answers is correct.

Question 11.

11. The number of telephone calls that arrive at a phone exchange is a random variable that follows a Poisson distribution. The expected number of calls per hour at the exchange is 6.

What is the probability that the exchange gets at least 1 call in the next hour? [7 pts]

- a) 0.0149
- b) 0.0174
- c) 0.9975
- d) 0.9881
- e) None of the answers is correct to the fourth decimal place.

Question 12.

12. A dice game is being played between two people. Each person rolls a fair die which has the numbers 1 to 6 on it. There are two cases in which Player 2 wins, and that is if the dice show up as a 'doubles' (such as 1-1 or 2-2) or if the sum of the dice is a 7.

What is the probability that Player 2 wins? [7 pts]

- a) $1/4$
- b) $1/3$
- c) $5/12$
- d) $5/6$
- e) None of the answers is correct.

Questions 13-14.

Suppose a box contains 10 red balls, 10 green balls, and 10 orange balls. You will be choosing 3 balls from the box without replacement.

13. What is the probability of drawing an orange on the first draw and a red on the second draw? [7 pts]
- a) 0.1111
 - b) 0.1000
 - c) 0.1034
 - d) 0.1149
 - e) None of the answers is correct to the fourth decimal place.

Question 14.

14. What is the probability of drawing an orange on the third draw? [7 pts]
- a) 0.1987
 - b) 0.3333
 - c) 0.4599
 - d) 0.6667
 - e) None of the answers is correct to the fourth decimal place.