

**22s:039 Probability and Statistics for Engineers**  
**Exam 2**  
**Form A**  
**Fall 2009**

Thursday, November 5, 8 - 10pm

100 possible points

Student Name -----

Section [letter/#] -----

Section [day/time] -----

Instructions:

- 1) Make sure you have the correct number of pages. There are 7 pages and 13 questions. Most questions are worth 8 points. There is one yes/no question that is worth 4 points.
- 2) Please use a pencil.
- 3) Questions can be clarified, but no hints will be provided.
- 4) You have 2 hours to complete the exam.
- 5) Show your student ID to your TA when handing the exam in.
- 6) Please hand-in your formula sheet with your exam.
- 7) If not stated, round answers to four significant digits.
- 8) You may begin, good luck.

**Questions 1-3.**

Suppose random variables  $X$  and  $Y$  have the joint probability distribution below:

		X			
		0	1	2	3
Y	0	0.02	0.05	0.01	0.02
	1	0.08	0.06	0.02	0.06
	2	0.15	0.10	0.05	0.38

1. Find  $P(Y = 2)$ 
  - a) 0.08
  - b) 0.22
  - c) 0.68
  - d) 0.05
  - e) None of the answers is correct to the second decimal place.
2. Find  $E[Y|X = 0]$ 
  - a) 1.52
  - b) 0.15
  - c) 1.30
  - d) 1.60
  - e) None of the answers is correct to the second decimal place.
3. Are  $X$  and  $Y$  independent?
  - a) Yes
  - b) No

**Questions 4-5.**

Six washing machines are selected independently and at random from a large pool of manufactured washers. Each machine is inspected and classified as having no scratches, minor scratches, or major scratches. The probability that a machine has no scratches, minor scratches, and major scratches is 0.75, 0.15, and 0.10, respectively.

4. What is the probability that none or one of the selected machines have no scratches?
- a) 0.0812
  - b) 0.0046
  - c) 0.1780
  - d) 0.0002
  - e) None of the answers is correct to four decimal places.
5. Given that four of the six selected machines have no scratches, what is the probability that one of the remaining two machines has a minor scratch?
- a) 0.48
  - b) 0.79
  - c) 0.24
  - d) 0.18
  - e) None of the answers is correct to the second decimal place.

**Questions 6-7.**

Suppose  $X$  and  $Y$  have the joint probability density function

$$f_{XY}(x, y) = \frac{1}{18}x^2y \quad \text{for} \quad 0 < x < 3 \quad \text{and} \quad 0 < y < 2.$$

6. Find  $P(X < 1.5)$

a) 0.1250

b) 0.1891

c) 0.8750

d) 0.5625

e) None of the answers is correct to four decimal places.

7. Find  $P(Y > 1|X < 1.5)$

a) 0.15

b) 0.64

c) 0.75

d) 0.56

e) None of the answers is correct to the second decimal place.

**Question 8.**

8. Suppose  $X$  has a lognormal distribution with parameters  $\theta = 0$  and  $\omega^2 = 4$ . Find  $P(10 < X < 50)$ .

a) 0.1001

b)  $\approx 0.0000$

c) 0.1175

d)  $\approx 1.0000$

e) None of the answers is correct to four decimal place.

**Question 9.**

9. Suppose we draw  $n = 5$  independent observations from a normal population with a mean of 80 and a variance of 7. Find  $P(\bar{X} < 78.11)$

a) 0.2358

b) 0.0548

c) 0.0885

d) 0.8554

e) None of the answers is correct to the four decimal places.

**Questions 10-11.**

Cracks in an oil pipeline follow a Poisson process at a rate of 0.8 cracks per mile.

10. What is the probability of three cracks occurring in a 5 mile stretch?
- a) 0.9093
  - b) 0.0383
  - c) 0.1954
  - d) 0.0907
  - e) None of the answers is correct to the second decimal place.
11. If you're inspecting the pipeline and start at a particular position, what is the probability that the first crack occurs between miles 2 and 3?
- a) 0.0586
  - b) 0.1390
  - c) 0.0469
  - d) 0.1112
  - e) None of the answers is correct to the four decimal places.

**Questions 12-13.**

Let  $X$  be the quantity of cell phone minutes my family uses each month. From past history, I know that  $X$  has the probability density function:

$$f(x) = \frac{1}{60,000} \left( \frac{-1}{600}x^2 + x \right) \quad \text{for} \quad 0 \leq x \leq 600$$

12. How many minutes do I expect my family to use during a month? (i.e. what is  $E(X)$ )

a) 450

b) 400

c) 300

d) 225

e) None of the answers is correct to the second decimal place.

13. In months where my family uses more minutes, my cell phone company charges me less per minute. Specifically, the monthly cost function,  $C(x)$ , in dollars with respect to the number of minutes,  $x$ , used each month is:

$$C(x) = 2x^{1/2}$$

What is the expected cost of my family using the cell phone in a month? (round to nearest dollar).

a) 45

b) 34

c) 30

d) 28

e) None of the answers is correct to the second decimal place.