Solutions for extra problems for topics since exam 2

8-5. a) Find n for the length of the 95% CI to be 40. $Z_{a/2} = 1.96$

$1/2 \text{ length} = (1.96)(20)/\sqrt{n} = 20$

$39.2 = 20\sqrt{n}$

$$n = \left(\frac{39.2}{20}\right)^2 = 3.84$$

Therefore, $n = 4$.

b) Find n for the length of the 99% CI to be 40. $Z_{a/2} = 2.58$

$1/2 \text{ length} = (2.58)(20)/\sqrt{n} = 20$

$51.6 = 20\sqrt{n}$

$$n = \left(\frac{51.6}{20}\right)^2 = 6.66$$

Therefore, $n = 7$.

8-78 $\mu = 50$ σ unknown

a) $n = 16$ $\bar{x} = 52$ $s = 1.5$

$$t_o = \frac{52-50}{8/\sqrt{16}} = 1$$

The *P*-value for $t_0 = 1$, degrees of freedom = 15, is between 0.1 and 0.25. Thus we would conclude that the results are not very unusual.

b) $n = 30$

$$t_o = \frac{52-50}{8/\sqrt{30}} = 1.37$$

The P-value for $t_0 = 1.37$, degrees of freedom = 29, is between 0.05 and 0.1. Thus we conclude that the results are somewhat unusual.

c) $n = 100$ (with $n > 30$, the standard normal table can be used for this problem)

$$z_o = \frac{52-50}{8/\sqrt{100}} = 2.5$$

The P-value for $z_0 = 2.5$, is 0.00621. Thus we conclude that the results are very unusual.

d) For constant values of $\bar{x}$ and s, increasing only the sample size, we see that the standard error of $\bar{X}$ decreases and consequently a sample mean value of 52 when the true mean is 50 is more unusual for the larger sample sizes.

8-80 a) The data appear to follow a normal distribution based on the normal probability plot since the data fall along a straight line.

b) It is important to check for normality of the distribution underlying the sample data since the confidence intervals to be constructed should have the assumption of normality for the results to be reliable (especially since the sample size is less than 30 and the central limit theorem does not apply).

c) No, with 95% confidence, we can not infer that the true mean could be 14.05 since this value is not contained within the given 95% confidence interval.

d) As with part b, to construct a confidence interval on the variance, the normality assumption must hold for the results to be reliable.

e) Yes, it is reasonable to infer that the variance could be 0.35 since the 95% confidence interval on the variance contains this value.

f) i) & ii) No, doctors and children would represent two completely different populations not represented by the population of Canadian Olympic hockey players. Because neither doctors nor children were the target of this study or part of the sample taken, the results should not be extended to these groups.

8-90 a) 95% Confidence Interval on the population proportion

n=1200 x=8 $\hat{p} = 0.0067$ $z_{\alpha/2}=z_{0.025}=1.96$

$$\hat{p} - z_{a/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{a/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.0067 - 1.96\sqrt{\frac{0.0067(1-0.0067)}{1200}} \le p \le 0.0067 + 1.96\sqrt{\frac{0.0067(1-0.0067)}{1200}}$$

$$0.0021 \le p \le 0.0113$$

9-102

	n	Test statistic	P-value	conclusion
a.	50	$z_0 = \frac{0.095-0.10}{\sqrt{0.10(1-0.10)/50}} = -0.12$	0.4522	Do not reject H_0

9-107 a) Rejecting a null hypothesis provides a *stronger conclusion* than failing to reject a null hypothesis. Therefore, place what we are trying to demonstrate in the alternative hypothesis.

Assume that the data follow a normal distribution.

b) 1) the parameter of interest is the mean weld strength, μ.
2) $H_0 : \mu = 150$
3) $H_1 : \mu > 150$
4) Not given
5) The test statistic is:

$$t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

6) Since no critical value is given, we will calculate the P-value
7) $\bar{x} = 153.7$, s= 11.3, n=20

$$t_0 = \frac{153.7 - 150}{11.3 / \sqrt{20}} = 1.46$$

P-value = $P(t \geq 1.46) = 0.05 < P - value < 0.10$

8) There is some modest evidence to support the claim that the weld strength exceeds 150 psi.
If we used $\alpha = 0.01$ or 0.05, we would not reject the null hypothesis, thus the claim would not be supported. If we used $\alpha = 0.10$, we would reject the null in favor of the alternative and conclude the weld strength exceeds 150 psi.

9-107 a) Rejecting a null hypothesis provides a *stronger conclusion* than failing to reject a null hypothesis. Therefore, place what we are trying to demonstrate in the alternative hypothesis.

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10-63 a) 1) The parameter of interest is the mean weight loss, μ_d
where d_i = Initial Weight − Final Weight.

2) $H_0 : \mu_d = 3$

3) $H_1 : \mu_d > 3$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\overline{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,7} = 1.895$.

7) $\overline{d} = 4.125$

$s_d = 1.246$

$n = 8$

10-68 a) 1) The parameters of interest are the proportions of those residents who wear a seat belt regularly, p_1 , p_2

2) $H_0 : p_1 = p_2$

3) $H_1 : p_1 \neq p_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ or $z_0 > z_{\alpha/2}$ where $z_{0.025} = 1.96$

7) $\hat{p}_1 = \frac{x_1}{n_1} = \frac{165}{200} = 0.825$ $\qquad$ $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = 0.807$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{198}{250} = 0.792$

$$z_0 = \frac{0.825 - 0.792}{\sqrt{0.807(1 - 0.807)\left(\frac{1}{200} + \frac{1}{250}\right)}} = 0.8814$$

8) Because $-1.96 < 0.8814 < 1.96$ do not reject H_0. There is not sufficient evidence that there is a difference in seat belt usage at $\alpha = 0.05$.

b) 1) The parameter of interest is the difference in mean volumes, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \delta}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject H_0 if $t_0 < -t_{\alpha/2,\nu}$ or $z_0 > t_{\alpha/2,\nu}$ where $t_{\alpha/2,\nu} = t_{0.025,18} = 2.101$

7) $\overline{x}_1 = 752.7$ $\quad \overline{x}_2 = 755.6$ $\qquad s_p = \sqrt{\frac{9(1.252)^2 + 9(0.843)^2}{18}} = 1.07$

$s_1 = 1.252$ $\quad s_2 = 0.843$

$n_1 = 10$ $\quad n_2 = 10$

$$t_0 = \frac{(752.7 - 755.6)}{1.07\sqrt{\frac{1}{10} + \frac{1}{10}}} = -6.06$$

8) Because $-6.06 < -2.101$, reject H_0 and conclude there is a significant difference between the two wineries with respect to mean fill volumes at a 5% significance level.