Practice Test 1 Answers

Answers available online at

`http://www.divms.uiowa.edu/~stoeckel/BusinessMathResources.html`

Tuesday, February 19, 2008

Mathematics for Business 22M:013:SCA TTh 6:00 - 8:00 PM

Instructor: Matt Stoeckel **Name:**

You have 60 minutes to complete your work. Show all work!

1. Simplify each expression.

   (a)
   $$\frac{5^{-1/2} \cdot 5x^{5/2}}{(5x)^{3/2}}$$

   Answer:

   $$= \frac{5^{-1/2} \cdot 5^1 x^{5/2}}{5^{3/2}x^{3/2}}$$
   $$= \frac{5^{1-1/2} \cdot x^{5/2}}{5^{3/2}x^{3/2}}$$
   $$= \frac{5^{1/2} \cdot x^{5/2}}{5^{3/2}x^{3/2}}$$
   $$= 5^{1/2-3/2}x^{5/2-3/2}$$
   $$= 5^{-1}x^{2/2} = x/5.$$

   (b)
   $$\sqrt{\frac{32a^4}{b^2}}$$

   Answer:

   $$= \frac{\sqrt{32a^4}}{\sqrt{b^2}} = \frac{\sqrt{2 \cdot 16a^4}}{\sqrt{b^2}} = \frac{4a^2\sqrt{2}}{|b|}.$$

   Why the absolute value in the denominator? Note that since $\sqrt{x} \geq 0$ we can always write $(\sqrt{x})^2 = x$. On the other hand, for real b, we have $b^2 \geq 0$ so we must write $\sqrt{b^2} = |b|$. Check that this is necessary, by using $b = -1$ or setting b to negative values and checking formula.

2. For each, find all solutions of the equation. Check your solutions in the original equation.

   (a) $\sqrt{x-10} - 3 = 0$

   Answer: Isolate radical: $\sqrt{x-10} = 3$.
   Square both sides: $x - 10 = 9$.
   Solve: $x = 19$.

   (b) $(5x+1)^2 - 16 = 0$

   Answer:
   Factor (difference of squares): $((5x+1) - 4)((5x+1) + 4) = 0$
   Simplify: $(5x-3)(5x+5) = 0$.
   By the zero factor property **either** $5x - 3 = 0$ **or** $5x + 5 = 0$.
   In the first case, $5x = 3$ so $x = 3/5$ and in the second case $5x = -5$ so $x = -1$.
   There are thus two solutions, $\{3/5, -1\}$.

3. Solve the equation by extracting square roots.

$$(3x-1)^2 = 64$$

   Answer:

$$(3x-1) = \pm\sqrt{64} \qquad \text{(note the } \pm\text{!)}$$
$$(3x-1) = \pm 8$$

   The equation $3x - 1 = -8$ solves as $3x = -7$ or $x = -7/3$, whereas $3x - 1 = 8$ solves as $3x = 9$ or $x = 3$. We thus have two solutions, $\{-7/3, 3\}$.

4. Use the Quadratic Formula to solve the equation.

$$2x^2 + 4x + 1 = 0$$

   Answer:

$$ax^2 + bx + c = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   Here, $a = 2, b = 4, c = 1$, thus,

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} \Rightarrow$$
$$x = \frac{-4 \pm \sqrt{8}}{4} \Rightarrow$$
$$x = \frac{-4 \pm 2\sqrt{2}}{4} = -1 \pm \sqrt{2}/2.$$

5. In each of the following, solve the given inequality. Write your solution in interval notation and sketch the solution on the real number line.

   (a) $6x - 4 \leq 2 + 8x$

      Answer:
      $\Rightarrow 6x - 4 - 6x \leq 2 + 8x - 6x$
      $-4 \leq 2 + 2x \Rightarrow$
      $-6 \leq 2x \Rightarrow$
      $-3 \leq x.$

   (b) $0 < 2x + 3 < 9$

      Answer: $\Rightarrow 0 - 3 < 2x + 3 - 3 < 9 - 3$
      $\Rightarrow -3 < 2x < 6 \Rightarrow -3/2 < x < 3.$

   (c) $|x - 5| \leq 5$

      Answer: $\Rightarrow -5 \leq x - 5 \leq 5$
      $\Rightarrow 0 \leq x \leq 10.$

6. Show that the points $(2, 1)$, $(4, 0)$ and $(5, 7)$ are the vertices of a right triangle.

   Answer: For this to be true the square of the length of one of the sides must be the sum of the squares of the lengths of the other two sides. We thus need to find the squares of the lengths of the sides to check.

   The square of the length of the side containing $(4, 0)$ and $(2, 1)$ is $(2 - 4)^2 + (1 - 0)^2 = 5$.
   The square of the length of the side containing $(4, 0)$ and $(5, 7)$ is $(5 - 4)^2 + (7 - 0)^2 = 50$.
   The square of the length of the side containing $(2, 1)$ and $(5, 7)$ is $(5 - 2)^2 + (7 - 1)^2 = 45$.
   Since $50 = 5 + 45$ we have that the square of the length of one of the sides is the sum of the squares of the lengths of the other two sides and thus the triangle is a right triangle.

7. In the following, write the standard form of the equation of the circle with the given characteristics.

   (a) Center: $(-7, -4)$; radius: 7

      Answer:

$$(x + 7)^2 + (y + 4)^2 = 49.$$

   (b) Center: $(-1, 2)$; solution point: (0,0)

      Answer: Since $(0, 0)$ is a point on the circle the *square* of the distance between $(0, 0)$ and $(-1, 2)$ is also the square of the radius. Thus the square of the radius is $(-1 - 0)^2 + (2 - 0)^2 = 5$. We thus have the center of the circle and the square of its radius. Thus the circle has standard form

$$(x + 1)^2 + (y - 2)^2 = 5.$$

8. Complete the table. Use the resulting solution points to sketch the graph of the equation,

$$y = 5 - x^2$$

Answer:

x	-2	-1	0	1	2
y	1	4	5	4	1
(x, y)	(-2,1)	(-1,4)	(0,5)	(1,4)	(2,1)

Clearly draw the graph and label your points!

9. Give an equation of the line that passes through the points $(-1, 3)$ and $(4, 4)$.

Answer: Labeling $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (4, 4)$. The slope of the line is

$$m = \frac{y_2 - y_2}{x_2 - x_1} = \frac{4 - 3}{4 - (-1)} = \frac{1}{5}.$$

It is now easy to use the point-slope form of the equation of a line,

$$y - y_1 = m(x - x_1),$$

by just substituting in the values we have for x_1, y_1, and m, we obtain the solution:

$$y - 3 = \frac{1}{5}(x + 1).$$

10. Write the slope-intercept form of the equation of the line passing through the given point and perpendicular to the given line.

Point $= (2, 1)$, *Line:* $4x - 2y = 3$

Answer: Note that we can obtain the slope of the line by putting it into slope-intercept form: $-2y = -4x + 3 \Rightarrow y = 2x - 3/2$, so the slope of the given line is 2. The slope of the perpendicular line is the opposite of the reciprocal of the slope equal to 2. Thus the slope of the perpendicular line is -1/2. Now use the point-slope form of a line with $m = -1/2$ passing through the point $(2, 1)$ to obtain:

$$y - 1 = (-1/2)(x - 2).$$